UG-A-440	**BMC-11**

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS WITH COMPUTER APPLICATION

FIRST YEAR

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Findy_n, when $y = \frac{x^2}{(x+1)^2(x+2)}$.

2. If $z = f(x, y)$ and $x = rcos\theta, y = rsin\theta$ prove that
$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

3. Find the envelope of the family of straight lines $y + tx = 2at + at^2$, where t is the parameter.

4. Find the value of $\int_0^a \int_0^x (x^2 + y^2)dydx$

5. Obtain reduction for $\int_o^n tan^n x dx; n \in N$

6. Prove that $\lim_{n\to\infty} \frac{x^n}{n!} = 0$

7. Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$

8. Test the convergence of $\sum \frac{2^n n!}{n^n}$

PART – B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. If $y = \mathrm{a}\cos(logx) + bsin(logx)$ prove that
$x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$.

10. Investigate the maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$.

11. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

12. Find the reduction formula for $I_n = \int x^m (logx)^n dx; m, n \in N$ and using this find the value of $\int x^4 (logx)^3 dx$

13. Find the volume of the cylinder $x^2 + y^2 = a^2$ above the xy-plane cut by the plane $x + y + z = a$.

14. Show that the sequence $\left(1 + \frac{1}{n}\right)^n$ converges.

15. Discuss the convergence of the series $\sum \frac{\sqrt{n+1-\sqrt{n}}}{n^p}$

16. Test the convergence of $\sum \frac{n^3+a}{2^n+a}$

U.G. DEGREE EXAMINATION - JUNE 2021
MATHEMATICS WITH COMPUTER APPLICATIONS
FIRST YEAR

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all FIVE questions.

1. State Euler's theorem (without proof).

2. Find the envelop of the family of a straight lines $y + tx = 2at + at^3$, the parameter being t.

3. Define the beta function and gamma function.

4. Prove that $lim_{n\to\infty} \frac{1}{2^n} = 0$.

5. Test the convergence of the series $\sum \frac{1}{(logn\,)^n}$ by root test.

PART - B **(4 x 5 = 20 Marks)**

Answer any FOUR questions.

6. Find the n^{th} differential coefficient of $cos^5\theta \cdot sin^7\theta$.

7. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

8. Find the area of the surface of the sphere of radius r.

9. Show that the sequence $\left(1 + \frac{1}{n}\right)^n$ converges.

10. Prove that the series $\sum \frac{1}{n^p}$ converges if $p > 1$ and divergies if $p \leq 1$.

11. State and prove Cauchy's second limit theorem.

12. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where $m, n > 0$.

PART - C **(4 x 10 = 40 Marks)**

Answer any FOUR questions.

13. Discuss the maxima and minima of the function $x^3y^2(6-x-y)$.

14. (i) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$ is $4acos\frac{\theta}{2}$

(ii) Show that the chord of curvature through the focus of a parabola is four times the focal distance of the point and the chord of curvature parallel to the axis has the same length.

15. A plane lamina of non-uniform density is in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If the density at any point (x,y) be Kxy, where K is a constant, then find the co-ordinates of the centroid of the lamina.

16. If the sequences (a_n) and (b_n) converge to 0 and (b_n) is strictly monotonic decreasing then prove that $lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = lim_{n\to\infty}\left(\frac{a_n-a_{n+1}}{b_n-b_{n+1}}\right)$provided the limit on the right hand side exists whether finite or infinite.

17. State and prove comparison test in detail. Use comparison test to prove that the series $\sum\frac{1^2+2^2+\cdots+n^2}{n^4+1}$ is a divergent series.

18. If $(a_n)\to a$ and $a_n > 0$, $a > 0$ for all n then prove that (i) $\left(\frac{1}{a^n}\right)\to\frac{1}{a}$ and (ii) $\sqrt{a_n}\to\sqrt{a}$

19. (i) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

(ii) Change the order of integration in the integral $\int_0^a\int_{x^2/a}^{2a-x} xy\,dx\,dy$ and evaluate it.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

FIRST YEAR

TRIGNOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. If $\tan(a+ib) = x+iy$, prove that $\frac{x}{y} = \frac{\sin 2a}{\sinh 2b}$.

2. If $i^{a+ib} = a+ib$, prove that $a^2+b^2 = e^{-(4n+1)\pi b}$.

3. Find the equation of plane passing through (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

4. Show that the lines $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and $\frac{x-5}{2} = \frac{y-8}{3} = \frac{z-7}{2}$ are coplanar and find the equation of the plane containing them.

5. Find the equation of the sphere passing through the points (0,0,0), (1,0,0), (0,1,0) and (0,0,1).

6. Show that $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

7. Obtain the directional derivative of $\varphi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$.

8. Using Green's theorem evaluate $\int_c (xy - x^2)dx + x^2 y dy$ along the closed curve C formed by $y=0, x=1$ and $y=x$.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Prove that $cos 8\theta = 128 cos^8\theta - 256 cos^6\theta + 160 cos^4\theta - 32 cos^2\theta + 1$

10. Sum to infinity the series $cos\alpha + \frac{1}{2} cos(\alpha+\beta) + \frac{1.3}{2.4}\cos(\alpha+2\beta) + \cdots\infty$

11. Find the shortest distance between the lines
$2x-2y+3z-12=0=2x+2y+z$ and $2x-z=0=5x-2y+9$.

12. Find the image of the point (1,3,4) under the reflection in the plane $2x - y + z + 3 = 0$. Hence, prove that the image of the line $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-4}{-3}$ is $\frac{x+3}{1} = \frac{y-5}{-5} = \frac{z-2}{-10}$.

13. Show that the plane $2x - 2y + z + 12 = 0$ touched the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Also, find the point of contact.

14. Prove: $\nabla \times \left(\nabla \times \vec{F}\right) = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F}$.

15. Evaluate $\int\int_S \vec{f} \cdot \vec{n} dS$, where $\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

16. Verify Gauss divergence theorem for $\vec{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2; z = 0$ and $z = h$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS WITH COMPUTER APPLICATIONS

FIRST YEAR

TRIGNOMETRY, ANALYTICAL GEOMETRY (3d) AND VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions in 50 words
[All questions carry equal marks]

1. Write the expansion of $\tan\theta$ (without proof).

2. Find the equation of the plane passing through (1,1,0), (1,2,1) and $(-2, 2, -1)$.

3. Obtain the equation of the sphere having its centre at the point (6, −1, 2) and touching the plane $2x - y + 2z - 2 = 0$.

4. If $\boldsymbol{u}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k}$ and $\boldsymbol{v}(t) = X(t)\boldsymbol{i} + Y(t)\boldsymbol{j} + Z(t)\boldsymbol{k}$ then prove that
$$\frac{d}{dt}(\boldsymbol{u}\cdot\boldsymbol{v}) = \boldsymbol{u}\cdot\frac{dv}{dt} + \frac{du}{dt}\cdot\boldsymbol{v}.$$

5. Write the statement of Stokes theorem (without proof).

PART - B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions out of Seven questions in 150 words
[All questions carry equal marks]

6. Expand $sin^3\theta \cdot cos^5\theta$ in a series of sines of multiples of θ.

7. Prove that the plane $x + 35y - 10z - 156 = 0$ is a bisector of the angle between two planes one of which is $4x - 3y + 12z + 13 - 0$. Also find the equation of the other plane.

8. Find the equations of the tangent planes of the sphere $x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$ which are parallel to the plane $x - z = 0$.

9. Find the angle between the surfaces $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at (4, −3, 2).

10. Verify the Gauss divergence theorem for the function $\boldsymbol{f} = a(x + y)\boldsymbol{i} + a(y - x)\boldsymbol{j} + z^2\boldsymbol{k}$ over the hemisphere bounded by the *xoy* plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

11. Separate into real and imaginary parts of $tan^{-1}(x + iy)$.

12. Find the shortest distance and the equation of the line of shortest distance between the straight lines $\frac{x+3}{-4} = \frac{y-6}{6} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions out of Seven questions in 400 words
[All questions carry equal marks]

13. Expand $\cos 9\theta$ in powers of $\cos\theta$ and hence deduce that $cos\frac{\pi}{9}\cdot cos\frac{2\pi}{9}\cdot cos\frac{4\pi}{9} = \frac{1}{8}$.

14. Find the image of the point (1, 3, 4) under the reflection in the plane $2x - y + z = -3$. Hence prove that the image of the line $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-4}{-3}$ is $\frac{x+3}{1} = \frac{y-5}{-5} = \frac{z-2}{-10}$.

15. (i) Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 - 4 = 0, 2x + 4y + 6z - 1 = 0$ and having its centre on the plane $x + y + z = 6$.
(ii) Find the equations of the spheres which passes through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0; 2x + y + z - 4 = 0$ and touches the plane $3x + 4y - 14 = 0$.

16. Prove that (i) $\boldsymbol{curl(f+g)} = \boldsymbol{curl(f)} + \boldsymbol{curl(g)}$; (ii) grad $(\boldsymbol{f}\cdot\boldsymbol{g}) = \boldsymbol{f}\cdot\boldsymbol{curl(g)} + \boldsymbol{g}\ \boldsymbol{X}\ \boldsymbol{curl(f)} + (\boldsymbol{f}\cdot\boldsymbol{\nabla})\boldsymbol{g} + (\boldsymbol{g}\cdot\boldsymbol{\nabla})\boldsymbol{f}$

17. Verify Stokes theorem for $f = (2x - y)i - yz^2j - y^2zk$ where S in the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

18. Prove that the condition for two lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ to be coplanar is $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$. Use this result to prove the lines $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and $\frac{x-5}{2} = \frac{y-8}{3} = \frac{z-7}{2}$ are coplanar and find the equation of the plane containing them.

19. (i) Evaluate $\int_c f\cdot dr$ where $f = (x^2 + y^2)i - 2xy\,j$ and the curve C is the rectangle in the $x - y$ plane bounded by $y = 0$, $y = b$, $x = 0$, $x = a$.
(ii) Evaluate $\iint f\cdot n\,dS$ where $f = (x^3 - yz)i - 2x^2yj + 2k$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0, x = a, y = a$ and $z = a$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATION

FIRST YEAR

COMPUTER FUNDAMENTALS AND PC SOFTWARE

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions

1. What is an operating system? Explain.
2. Write a short note on parallel processing.
3. Write a short note on E-mail.
4. How do you set date and time on your computer?
5. Write short note on multimedia.
6. Explain varies steps in creating a new template in a Microsoft word document.
7. How do you insert a clip art in a Microsoft power point slide?
8. How do you include transition effects to a power point presentation?

PART - B **(5 x 10 = 50 Marks)**

Answer any FIVE questions

9. Explain different types of memory units used in computers.
10. Write a short note on computer security.
11. Explain disk defragmenter and disk cleanup features in Windows.
12. Briefly explain different parts of a Window.
13. Explain different page formatting tools that are available in Microsoft word.
14. How do you include mail merge in a Microsoft word document?
15. Explain inserting deleting a slide and changing slide background colour in Microsoft power point.
16. How do you work with charts in a Microsoft PowerPoint?

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

FIRSTYEAR

COMPUTER FUNDAMENTALS AND PC SOFTWARE

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5x2 = 10Marks)**

Answer any FIVE questions

1. What is system software? Enlist them.
2. Give the difference between WAN and MAN network.
3. What is a Recycle bin?
4. Difference between Header and footer in MS-Word.
5. What is Animation?

PART - B **(4 x 5 =20 Marks)**

Answer any FOUR questions

6. What is ROM? Describe its various types.
7. Distinguish between Asynchronous and synchronous transmissions.
8. Explain in brief the concept of working with windows.
9. Write the short note of page break and section break.
10. Write the steps to transitions to a slide in PowerPoint.
11. Explain various types of computers in detail.
12. What is meant by a cache memory explain?

PART - C **4 x 10 = 40 Marks)**

Answer any FOUR questions s

13. Explain in briefly the network and distributed operating system.
14. Write short notes on : a)BITNET b)ISDN c)NICNET
15. Define the term computer virus. Also explain their types.
16. How to Work with Character Formatting into a document? Explain.
17. Explain in detail about different views of a slide in PowerPoint.
18. Draw a block diagram of a computer. Explain the function of each of the blocks.
19. Explain various output devices in details.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS WITH COMPUTER APPLICATIONS

SECOND YEAR

GROUPS AND RINGS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions.

1. What is an equivalence relation? Show that if ρ and σ are equivalence relations defined on a set S. Prove that $\rho \cap \sigma$ is an equivalence relation.

2. Show that cube roots of unity with usual multiplication forms a group.

3. Define a Normal subgroup of a group and give an example.

4. Show that every cyclic group is abelian.

5. Let G be a group and let a be a fixed element of G. Let $H_a = \{x/x \in G \ \ and\ ax = xa\}$. Show that H_a is a subgroup of G.

6. Prove that the intersection of two subrings of a ring R is a subring of R.

7. If $f: R \rightarrow R^{'}$ is a Ring homomorphism and K is the kernel of f, Prove that K is an ideal of R.

8. Prove that any Field is an Euclidean domain if $d(a) = 1$ for all $a \in F - \{0\}$.

PART - B **(5 x 10 = 50 Marks)**

Answer any FIVE questions.

9. Let G deonote the set of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where $x \in R^*$ (non-zero real numbers). Prove that G forms a group under matrix multiplication.

10. State and prove Fundamental theorem of homomorphism.

11. Prove that every integral domain can be imbedded in a field.

12. State and prove Cayley's theorem.

13. State and Prove Lagrange's theorem.

14. Let R be a commutative ring with identity. Prove that an ideal M of R is maximal if and only if R/M is a field.

15. (i) Prove that every subgroup of an abelian group is normal.
(ii) Prove that isomorphism is an equivalence relation among groups.

16. Let ‘a’ be a non-zero element of an Euclidean domain R. Show that a is a unit in R if and m/y if $d(a) = d(I)$.

	UG-A-453	BMC-22

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

CLASSICAL ALGEBRA AND NUMERICAL METHOD

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$. Given that one of the roots is $1 - \sqrt{5}$.

2. Find the equation each of whose roots exceeds by 2, root of the equation $x^3 - 4x^2 + 3x - 1 = 0$.

3. Find the sum of the series to infinity $\log 2 - \frac{(log\,2)^2}{2!} + \frac{(log\,2)^3}{3!} + \cdots$

4. Find the sum of the series $\frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \frac{1}{5\cdot6} + \cdots\infty$

5. Find the approximate real root of the equation $xe^x - 3 = 0$. $1 < x < 1.1$ using Regula Falsi method.

6. Using Newton divided difference formula find the missing value using the table

x	1	2	4	5	6
y	14	15	5	?	9

7. Find $f^{'}(4)$ from the table

x	0	2	3	5
f(x)	8	6	20	108

8. Using Euler's method find $y(0.2)$ given that $y^{'} = x + y$ given that $y(0) = 1$

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Find the sum to infinity of the series $\frac{7}{72}+\frac{7.28}{72.96}+\frac{7.28.49}{72.96.120}+\cdots$.

10. Sum to infinity the series $\frac{2.3}{3!}+\frac{3.5}{4!}+\frac{4.7}{5!}+\frac{5.9}{6!}+\cdots$

11. Solve: $6x^5+11x^4-33x^3-33x^2+11x+6=0$

12. If α,β,γ are the roots of equation $x^3+px^2+qx+r=0$ find the value of
    (i) $\sum\alpha^2$
    (ii) $\sum\frac{1}{\alpha}$
    (iii) $\sum\frac{1}{\alpha\beta}$
    (iv) $\sum\alpha^2\beta^2$

13. Apply Gauss Jordan method to solve the system
$x+y+z=9$
$2x-3y+4z=13$
$3x+4y+5z=40$

14. Using Lagrange's formula fit a polynomial to the data:

x	-1	0	2	3
y	-8	3	1	12

and hence find y when $x=1$

15. Evaluate $\int_0^1\frac{1}{1+x^2}dx$ using Trapezoidal rule by taking h=0.25 and by taking h= 0.5.

16. Apply the fourth order Runge-Kutta method, to find an approximate value of y when $x=0.2$ given that $\frac{dy}{dx}=x+y, y(0)=1$.

U.G. DEGREE EXAMINATION – JUNE 2021

MATHS WITH COMPUTER APPLICATIONS
SECOND YEAR

PROGRAMMING IN C AND C++

Time: 3 Hours **Maximum Marks: 75**

SECTION - A **(5 × 5 = 25 Marks)**

Answer any FIVE questions.

1. Write the general structure of C program with an example.

2. Discuss C storage class with example.

3. Compare structure with unions.

4. Write a C program to copy the concept of one file to another file.

5. What is Object and Class? Explain with example.

6. Write a C program to find the biggest of the given n numbers.

7. What is recursion? Explain with example.

8. Explain the features of object oriented programming with example.

SECTION - B **(5 × 10 = 50 Marks)**

Answer any FIVE questions.

9. What are the various data types available in C? Explain the memory size and range of data possible.

10. What is an array? How a single dimension and two-dimension arrays are declared and initialized?

11. What is dynamic memory allocation? Write and explain the different dynamic memory allocation functions in C.

12. List and explain the important file handling functions in C with examples.

13. Explain different forms of inheritance. Illustrate with an example each type with an example.

14. Write a program to demonstrate how a static data is accessed by a static member function.

15. Explain copy constructor and destructor with suitable C++ coding.

16. What is friend function? What is the use of using friend functions in C++? Explain with a Program.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

THIRD YEAR

REAL AND COMPLEX ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Prove that $[0,1] = \{x/0 \leq x \leq 1\}$ is uncountable.

2. Prove if f is a continuous function from a metric space M_1 into a metric spaceM_2 and if M_1 is connected, then the range of f is also connected.

3. Find the Taylor series about $x = 2$ for
$$f(x) = x^3 + 2x + 1 \qquad (-\infty < x < \infty)$$

4. Prove that if A is a closed subset of the compact metric space $\langle M, \rho \rangle$ then metric space $\langle A, \rho \rangle$ is also compact.

5. Show that $f(z) = sinz$ is an analytic function.

6. Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

7. Find the invariant points of the transformation $w = \frac{2z+4i}{1+iz}$

8. Expand $f(z) = \frac{1}{z(z-1)}$ as Laurent's series valid in $|z| < 1$ and $< |z| < 2$.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Prove that the countable union of countable sets is countable.

10. Prove that if M is a compact metric space, then M has the Heine-Borel property.

11. State and Prove second fundamental theorem of calculus.

12. Prove that a subset of R is connected if and only if it is an interval.

13. Show that an analytic function with (i) constant real part is constant and (ii) constant modulus is constant.

14. State and prove Cauchy residue-theorem.

15. Find the bilinear transformation which maps the points $z_1 = \infty, z_2 = i, z_3 = 0$ on to the points $w_1 = 0, w_2 = i, w_3 = \infty$ respectively.

16. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} \, (a^2 < 1)$ using contour integration.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

THIRD YEAR

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Prove that the intersection of two subspaces of a Vector space is a subspace.

2. Prove in $V_3(R)$ the vectors (1,2,1), (2,1,0) and $(1, -1,2)$ are linearly independent.

3. Find the linear transformation $T: V_3(R) \rightarrow V_3(R)$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard basis.

4. Define norm of a vector $x \in V$, where V is an inner product space. Also, Prove $\|\alpha x\| = |\alpha|\|x\|$ for all $\alpha \in F, x \in V$.

5. If S is any subspace of an inner product space V then $S^{\perp}$ is a subspace of V

6. Let f be the bilinear form defined on $V_2(R)$ by $f(x,y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f with respect to the basis $\{(0,1), (1,0)\}$.

7. Prove that in any distributive lattice if $x \vee a = y \vee a$ and $x \wedge a = y \wedge a$ then $x = y$.

8. Define a Lattice. Give two examples.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Let V be a finite dimensional vector space over F and W be a subspace of V, then prove that $dimW \leq dimV$ and $dim\frac{V}{W} = dimV - dimW$.

10. Let V and W be two finite dimensional vector spaces over F. Let $dimV = m$ and $dimW = n$. Then Prove that $L(V,W)$ is a vector space of dimension mn over F.

11. Prove that every finite dimensional inner product space has an orthonormal basis.

12. Let V be a vector space over a field F such that $S, T \subseteq V$ then prove the following:
(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ (ii) $L(S \cup T) = L(S) + L(T)$

13. If V is finite dimensional inner product space and W is a subspace of V then prove that $V = W \oplus W^{\perp}$ (ie., V is the direct sum of W and $W^{\perp}$).

14. Reduce the Quadratic form to diagonal form

$$x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$$

15. If B is a Boolean algebra, then Prove
(i) $(a \vee b)' = a' \wedge b'$ (ii) $(a \wedge b)' = a' \vee b'$ (iii) $(a')' = a$

16. If L is a lattice and $a, b, c, d \in L$, then prove the following
(i) when $a \leq b$ and $c \leq d$ prove $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$
(ii) $a \vee a = a$
(iii) $a \wedge a = a$

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

THIRD YEAR

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time: 3 Hours **Maximum Marks: 75**

PART – A **(5 X 5 =25 Marks)**

Answer any FIVE of the following.

1. Use simplex method to maximize $Z = 5x_1+4x_2$
   subject to the constraints:
   $4x_1+5x_2 \le 10$, $3x_1+ 2x_2 \le 9$,
   $8x_1+ 3x_2 \le 12$, $x_1, x_2 \le 0$.

2. Show that the dual of the dual is the primal.

3. Explain Northwest-corner method in transportation problem.

4. Obtain the initial basic feasible solution to the following transposition problem by least cost method.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

5. For a game with the following pay-off matrix, determine the optimal strategies and the value of the game.

$$\begin{array}{cc} & A \\ B & \begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix} \end{array}$$

6. A and Beach takes out one or two matches and guess how many matches opponent has taken. If one of the players guesses correctly then loser has to pay him as many rupees as the sum of the numbers held by both players. Otherwise, the payout is zero. Write down the pay-off matrix and obtain the optimal strategies of both the players.

7. A company uses rivets at a rate of 5,000 kg. per year, rivets costing Rs. 2 per kg. It cost Rs. 20 to place an order and the carrying cost of inventory is 10% per year. How frequently should order for rivets be placed and how much?

8. Customers arrive at a box office window, being manned by single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean 90 seconds. Find the average waiting time of a customer.

PART – B **(5 X 10 =50 Marks)**

Answer any FIVE of the following.

9. Use Big-M method to solve the following linear programming problem.

Maximize $z = 2x_1 + x_2 + 3x_3$

subject to the constraints

$x_1 + x_2 + 2x_3 \leq 5$, $2x_1 + 3x_2 + 4x_3 = 12$

and $x_1, x_2, x_3 \geq 0$.

10. Solve the following linear programming problem by dual simplex method.

Minimize $z = x_1 + 2x_2 + 3x_3$

subject to the constraints

$x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$

$x_2 - x_3 \geq 2$, and $x_1, x_2, x_3 \geq 0$.

11. Solve the following transportation problem to minimize the total cost, obtaining the initial solution by Vogel's approximation method. If the optimal solution is not unique, find an alternative optimum solution.

					Availabl e
	7	9	3	2	16
	4	4	3	5	14
	6	4	5	8	20
Requirement	11	9	22	8	

12. Five jobs are to be processed and five machines are available. Any machine can process any job with the resulting profit as follows.

Job	Machine				
	A	**B**	**C**	**D**	**E**
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

What is the maximum profit that may be expected if an optimum assignment is made?

13. Discuss the EOQ model with constant rate of demand and variable order cycle time where the shortages are allowed.

14. A supermarket has two sales girls running up sales at the counters if the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 an hour, (i) what is the probability of having to wait for service? (ii) what is the expected percentage of idle time for each girl? (iii) If a customer has to wait, what is the expected length of his waiting time?

15. Customers arrive at a box office window, being manned by single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find (i) the average waiting time of a customer, (ii) the average number of customers in the system and (iii) the average queue length.

16. Solve the game whose pay-off matrix is given below by graphical method.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 4 & -2 & 3 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 1 & -2 & 0 \end{bmatrix} \end{array}$$

UG-A-455 **BMC-34**

U.G. DEGREE EXAMINATION - JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

THIRD YEAR

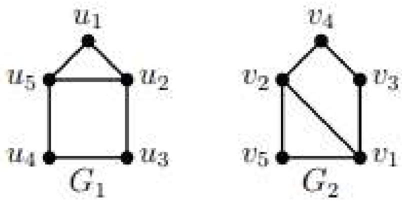
GRAPH THEORY

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5x5 = 25 Marks)**

Answer any FIVE questions.

1. When do you say two graphs are isometric? Check whether the given graphs are isometric



2. Prove that for any positive integer n, a tree with n vertices has n-1 edges.
3. State and prove handshaking theorem.
4. Show that an edge e(u;v) is not a cut-edge of G if and only if e belongs to a cycle in G.
5. Prove that Every hamiltonian graph is 2-connected.
6. For any graph G, $\chi(G) \leq 1+\max\delta(G')$ where the summation is taken over all induced subgraphs G'of G.
7. If G is uniquely n-colourable, then $\delta(G) \geq n - 1$.
8. Prove that in a digraph D, sum of all the in-degrees of all the vertices is equal to the sum of their out- degrees, each sum being equal to the number of arcs in D.

PART - B **(5x10 = 50 Marks)**

Answer any FIVE questions.

9. a) Let G_1 be a (p_1, q_1) and G2 a (p_2, q_2) graph. Prove the following

   1. $G_1 \cup G_2$ is a $(p_1 + p_2, q_1 + q_2)$ graph.

   2. $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph.

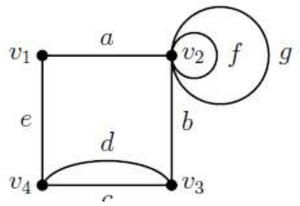
   3. $G_1 \times G_2$is a $(p_1p_2, q_1p_2 + q_2p_1)$ graph.

   b) Prove that A graph G with p points and $\delta \geq \frac{p-1}{2}2$ is connected.

10. a) Prove that with usual notations for any graph G ,$\kappa \leq \lambda \leq \delta$

    b) Prove that a graph G with p points and $\delta \geq \frac{p-1}{2}2$ is connected.

11. a) For the graph G find its adjacency matrix and incidence matrix.



b) Prove that a vertex v of a connected graph is a cut-vertex if and only if there exist vertices x and y ($\neq$ v) such that every (x; y)-path contains v.

12. Let G be a (p, q) graph. Prove that the following statements are equivalent.

(1) G is a tree.

(2) every two points of G are joined by a unique path.

(3) G is connected and $p = q + 1$

(4) G is acyclic and $p = q + 1$

13. a) Prove that every connected graph has a spanning tree. Also prove that for G be a (p, q) connected graph. Then $q \geq p - 1$

b) Let T be a spanning tree of a connected graph G. Let x = uv be an edge of G not in T. Then prove that T + x contains a unique cycle.

14. a) Prove that Every tree has a centre consisting of either one point or two adjacent points.

b)Let G be any graph. prove that the following statements are equivalent.

1. G is 2-colourable.
2. G is bipartite.

15. a)If G is k-critical, then prove that $\delta(G) \geq k - 1$ and Every k-chromatic graph has at least k vertices of degree at least $k - 1$.

b) For any graph G, prove that $\chi \leq \Delta + 1$.

16. a) If two digraphs are isomorphic then prove that the corresponding points have the same degree pair.

b) Prove that a weak digraph D is an Eulerian if every point of D has equal in-degree and out-degree.

UG-A-456	**BMC-35**

U.G. DEGREE EXAMINATION – JUNE 2021

MATHS WITH COMPUTER APPLICATIONS

THIRD YEAR

INTRODUCTION TO INTERNET PROGRAMMING (JAVA)

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 × 5 = 25 Marks)**

Answer any FIVE questions.

1. Explain the three principles of Java.
2. Write the difference between break and continue statements in Java.
3. What is an abstract class? Explain.
4. Explain the term package.
5. Explain in detail about life cycle of an applet.
6. Explain in brief the overloading methods in Java.
7. What are literals in Java? Mention their different types.
8. Explain the concepts JDK, JRE and JVM.

PART - B **(5 × 10 = 50 Marks)**

Answer any FIVE questions.

9. What are java buzz words? Give brief description.
10. Write a program to find a number provided by the user in a given array using binary search.
11. What is the purpose of the finally clause of a try-catch-finally statement?
12. What is an interface? How is it implemented?
13. What is the purpose of the wait (), notify(), and notify All() methods?
14. Describe all the primitive data types supported by Java with appropriate examples. Also specify their storage capacity/range.
15. Explain the concept of inheritance and its types.
16. What is method overriding and how does one prevent a method from being overridden?